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Reduced Mass-Weighted Proper Decomposition of an Experimental Non-Uniform Beam

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Purpose

Estimate the mode shapes from displacement measurements of a non-uniform beam.

TRAITS OF METHOD

Non- Uniform Beam

Reduced Order Mass Matrix

M sensors, N samples

Estimates for Linear Normal Modes (LNMs)

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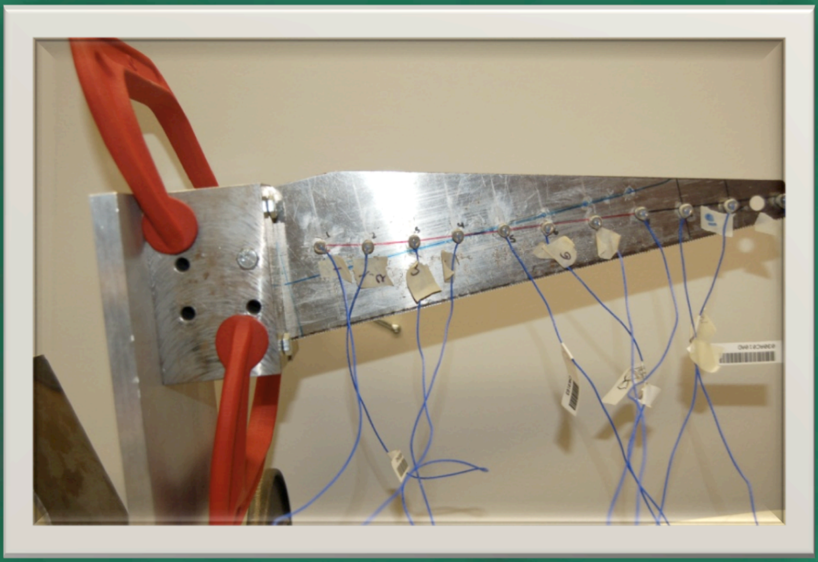
Experimental Setup

Thin non-uniform beam sensed with 11 accelerometers.

Accelerometers were placed in one inch intervals at the midpoint of that cross section.

Beam was aligned so that the line of accelerometers was horizontal



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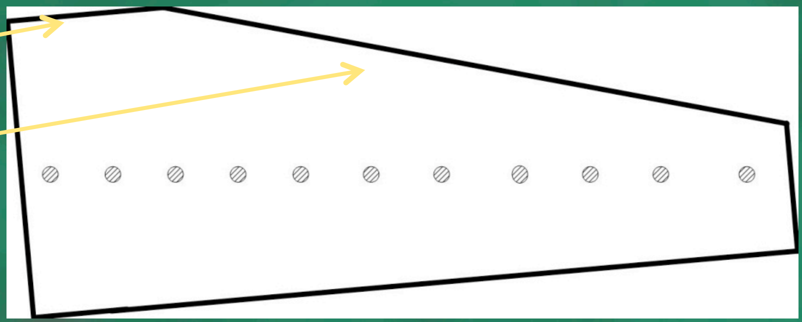
Beam Dimensions

Height: 3.5 inches
Length: 11.5 inches
Thickness: 1/32 inches
Top: 1.5 inches
Slant: 10.125 inches
Tip: 1 inch
Density: 0.284 lbs_m / in^3
Young's Modulus: 29×10^6 psi
Poisson's ratio: 0.313



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Procedure
Displacement Ensemble

$$X = \begin{bmatrix} x_1(0) & x_1(\Delta t) & \cdots & x_1(N\Delta t) \\ x_2(0) & x_2(\Delta t) & \cdots & x_2(N\Delta t) \\ \vdots & \vdots & \ddots & \vdots \\ x_M(0) & x_M(\Delta t) & \cdots & x_M(N\Delta t) \end{bmatrix}$$

Each row is filtered and the mean is subtracted. $X \in \mathbb{R}^{MxN}$

M = number of sensors, N = number of time samples

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Procedure
Compute Correlation Matrix R

$$R = \frac{1}{N} X X^T$$

If Mass matrix has the dimensions MxM then solve

$$R\mathcal{M}v = \lambda v$$

N = number of samples, M = number of sensors

$\mathcal{M}$= Mass matrix for an uniform beam

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POD vs MWPOD

POD - produces estimates of mode shapes when mass is uniform $Rv = \lambda v$

Mass Weighted POD – produces estimates of mode shapes when the mass is NOT uniform $R\mathcal{M}v = \lambda v$

Reduced Mass Weighted POD – produces estimates of mode shapes when the mass is NOT uniform AND the number of sensors is less than the number of DOF $R\mathcal{M}_r v = \lambda v$

ν_i eigenvectors estimate modes, λ_i estimate modal energies/amplitudes

Procedure
Reduced Mass Weighted Proper Decomposition

Solve $RM_r v = \lambda v$ where v correlates to estimates of LNMs (lightly damped, free vibration) and λ relates to the energy density of the corresponding modes.

The strategy of computing M_r is based on the method of interpolation between the known measured displacements (at the sensors) to approximate the unknown displacements (between sensors). Similar to FEM.

Procedure
Reduced Mass Matrix

If Mass matrix dimensions are larger than MxM then compute a Reduced Mass Matrix M_r whose dimensions are MxM using interpolating tent functions.

$$M_{r\,mn} = M_{r\,nm} = \int_0^L \rho(x)\,\eta_n \eta_m dx$$

Where $\rho(x)$ *is mass per unit length*, η_n *and* η_m are shape functions.

The above function is used for 1-D continuous beams.

Results

Natural Frequency	Experimental
1st	8.545 Hz
2nd	40.28 Hz
3rd	107.4 Hz
4th	205.1 Hz

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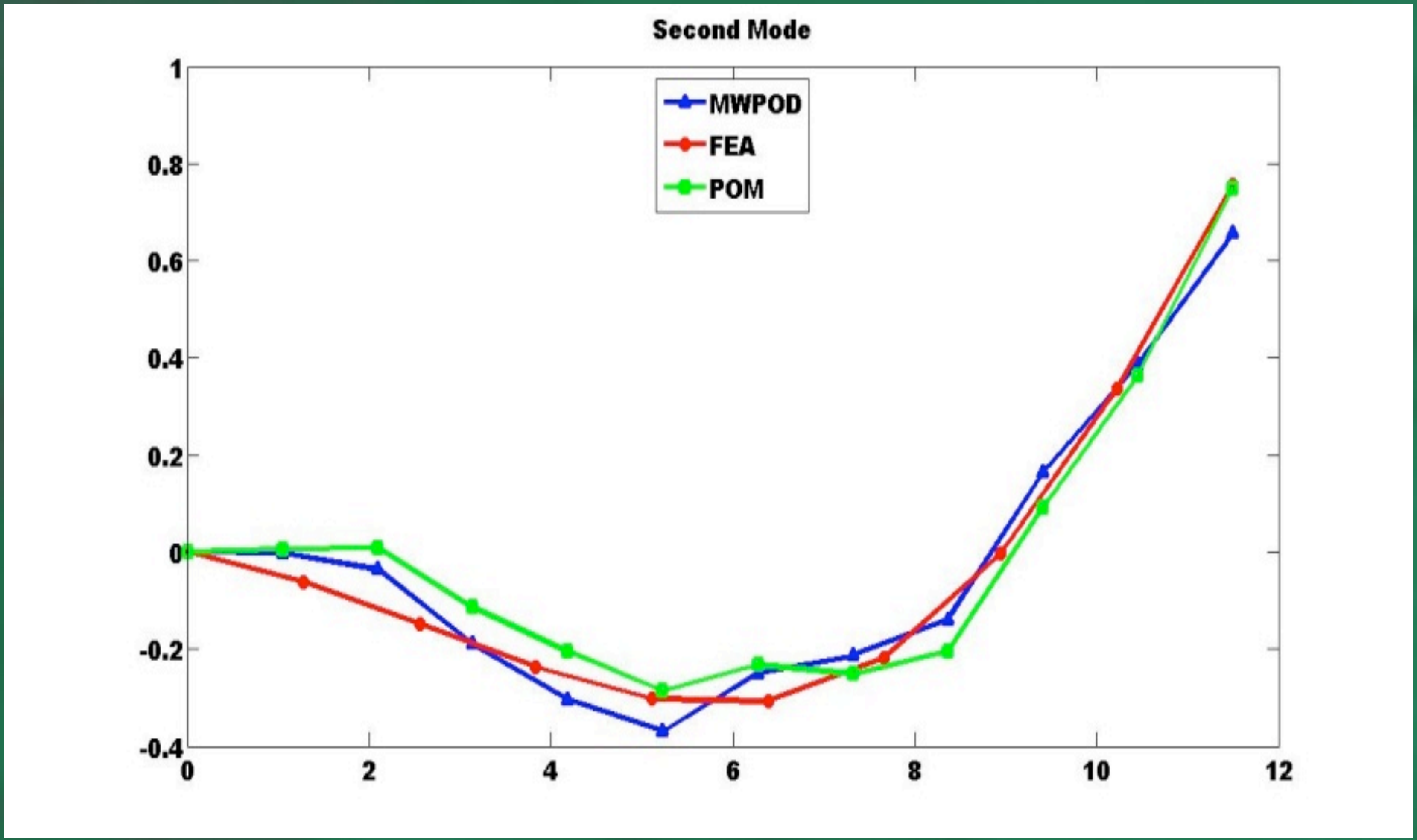
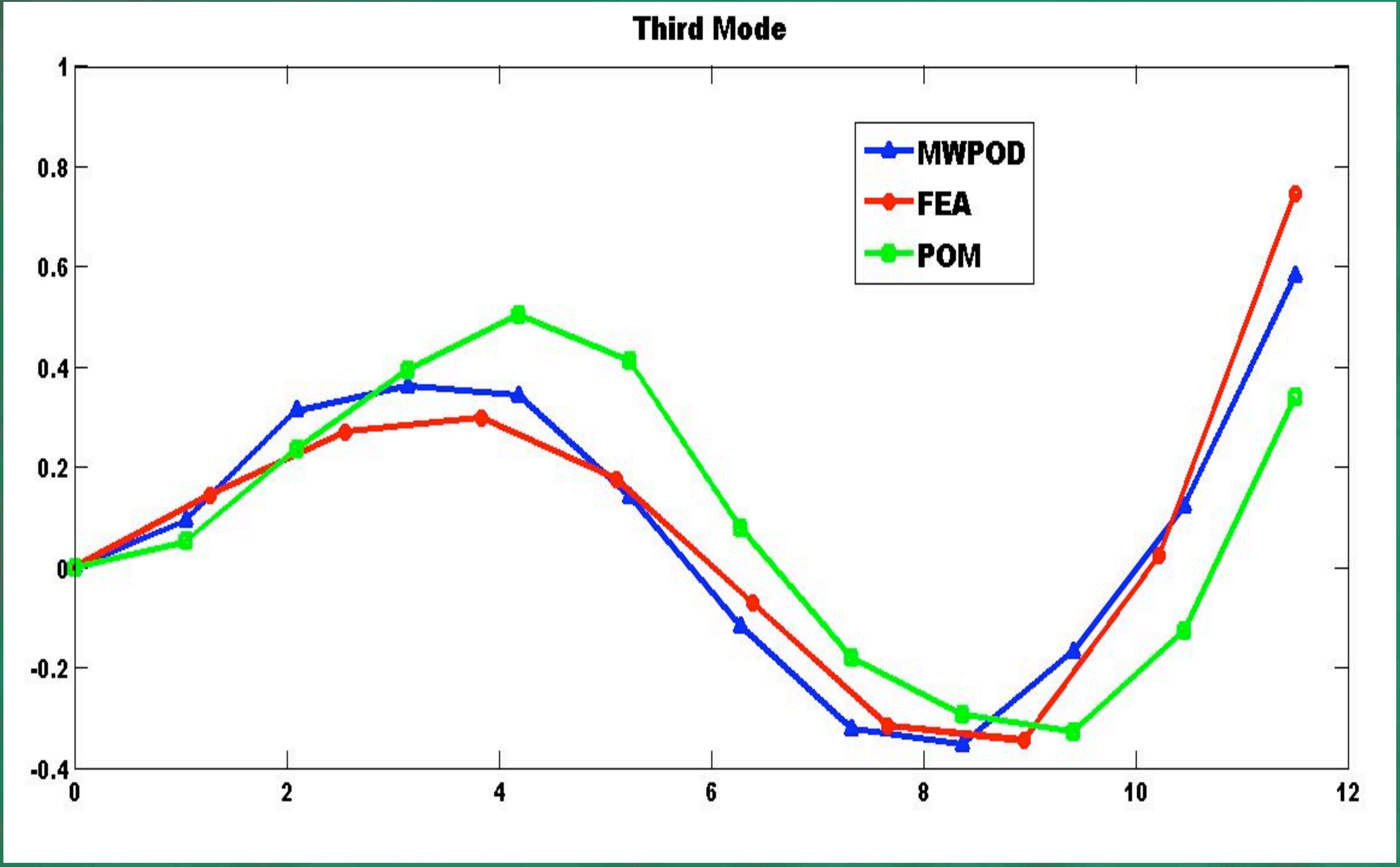
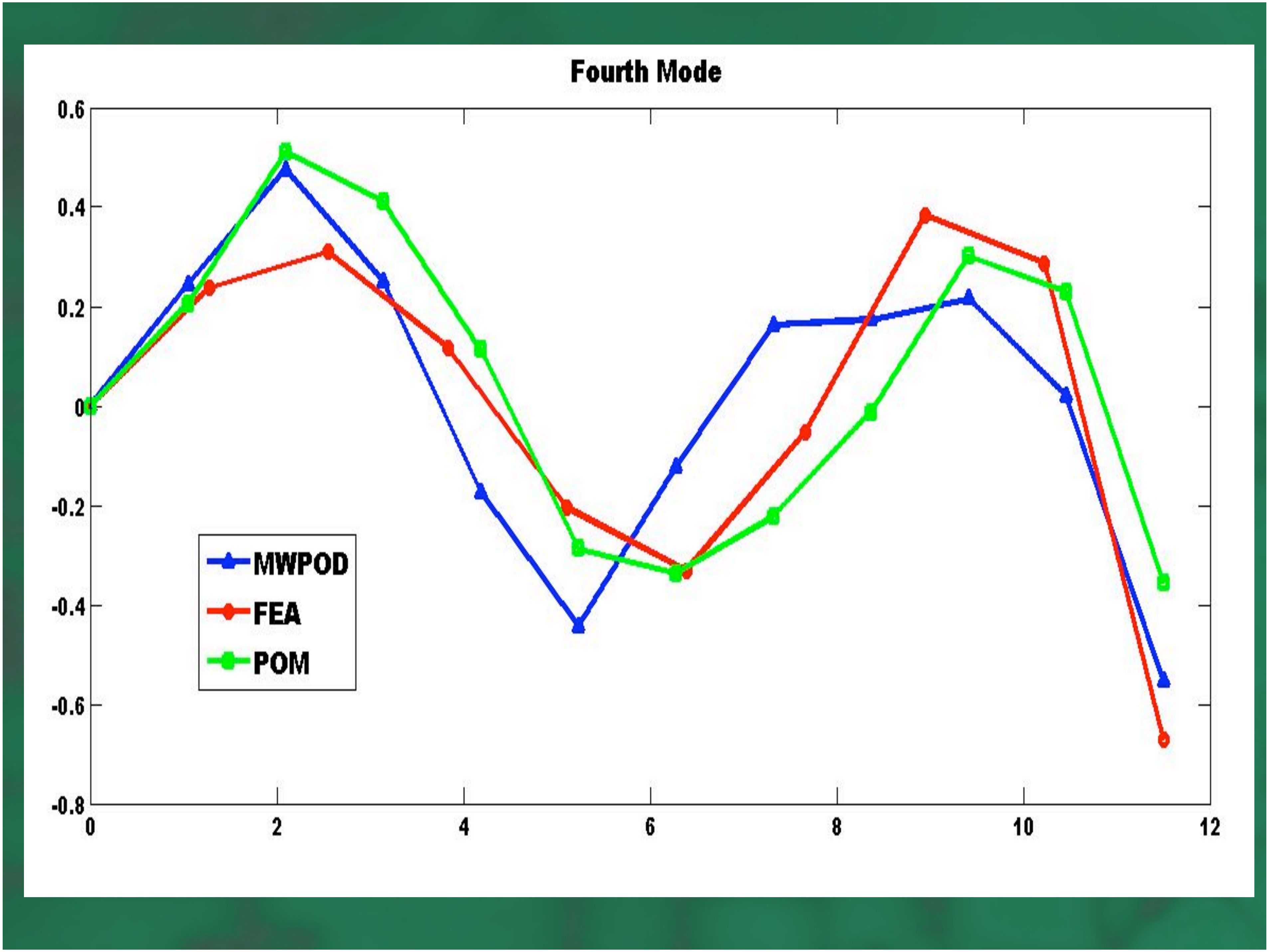
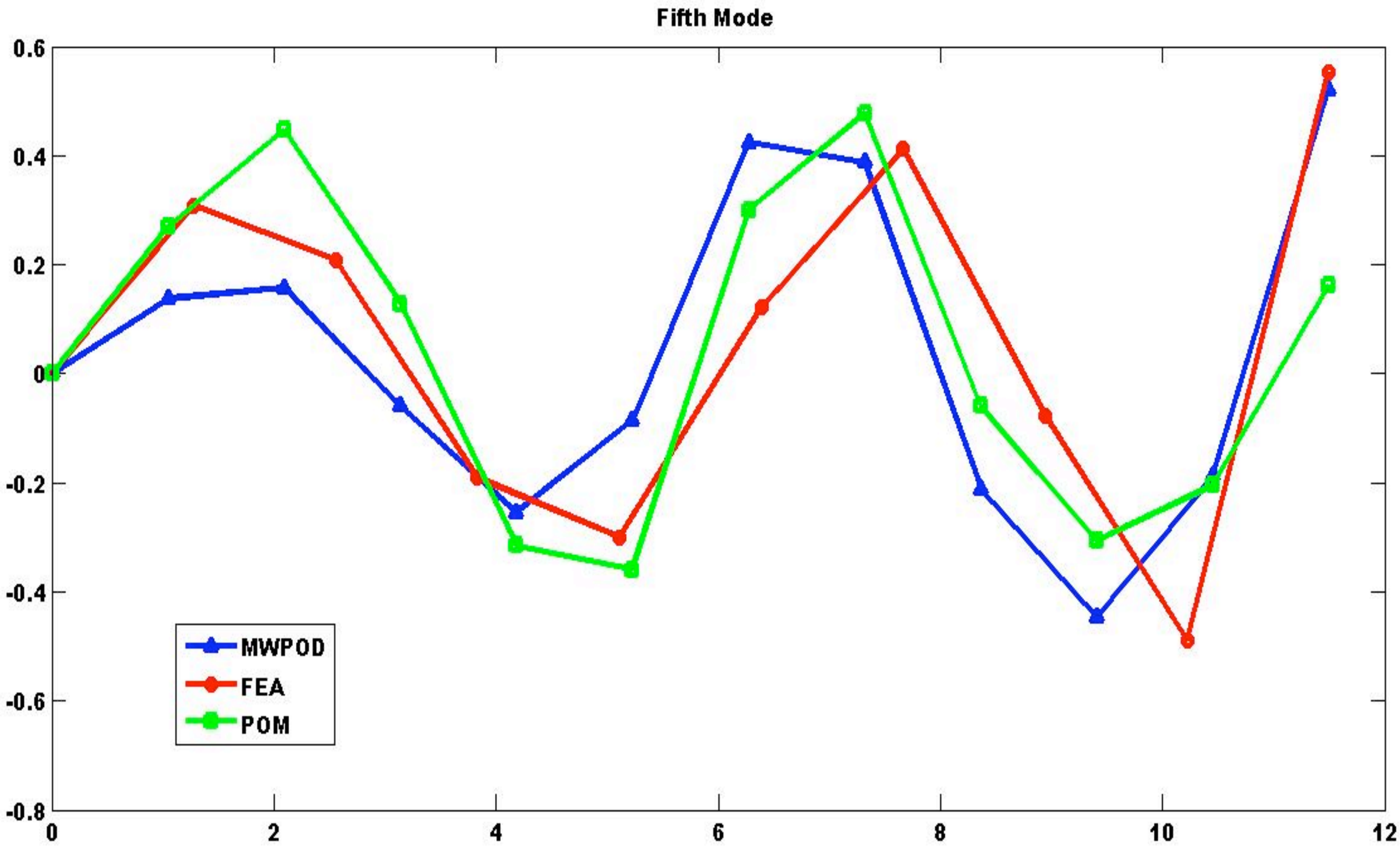
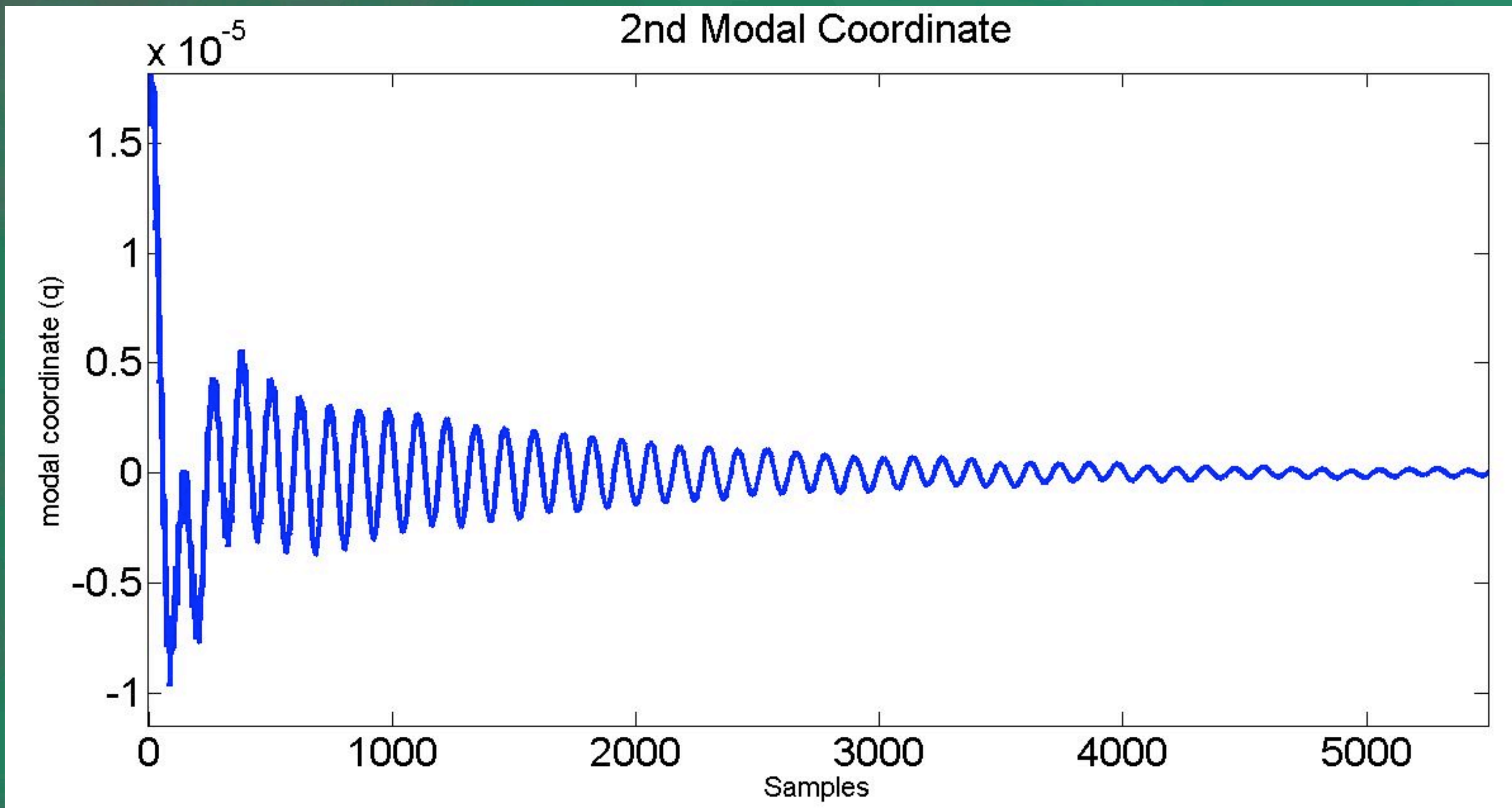



Third Mode
MWPOD
FEA
POM
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10
12



Fourth Mode
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
2
4
6
8
10
12
MWPOD
FEA
POM


Fifth Mode
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
2
4
6
8
10
12
MWPOD
FEA
POM


2nd Modal Coordinate
x 10^-5
1.5
1
0.5
0
-0.5
-1
modal coordinate (q)
0
1000
2000
3000
4000
5000
Samples

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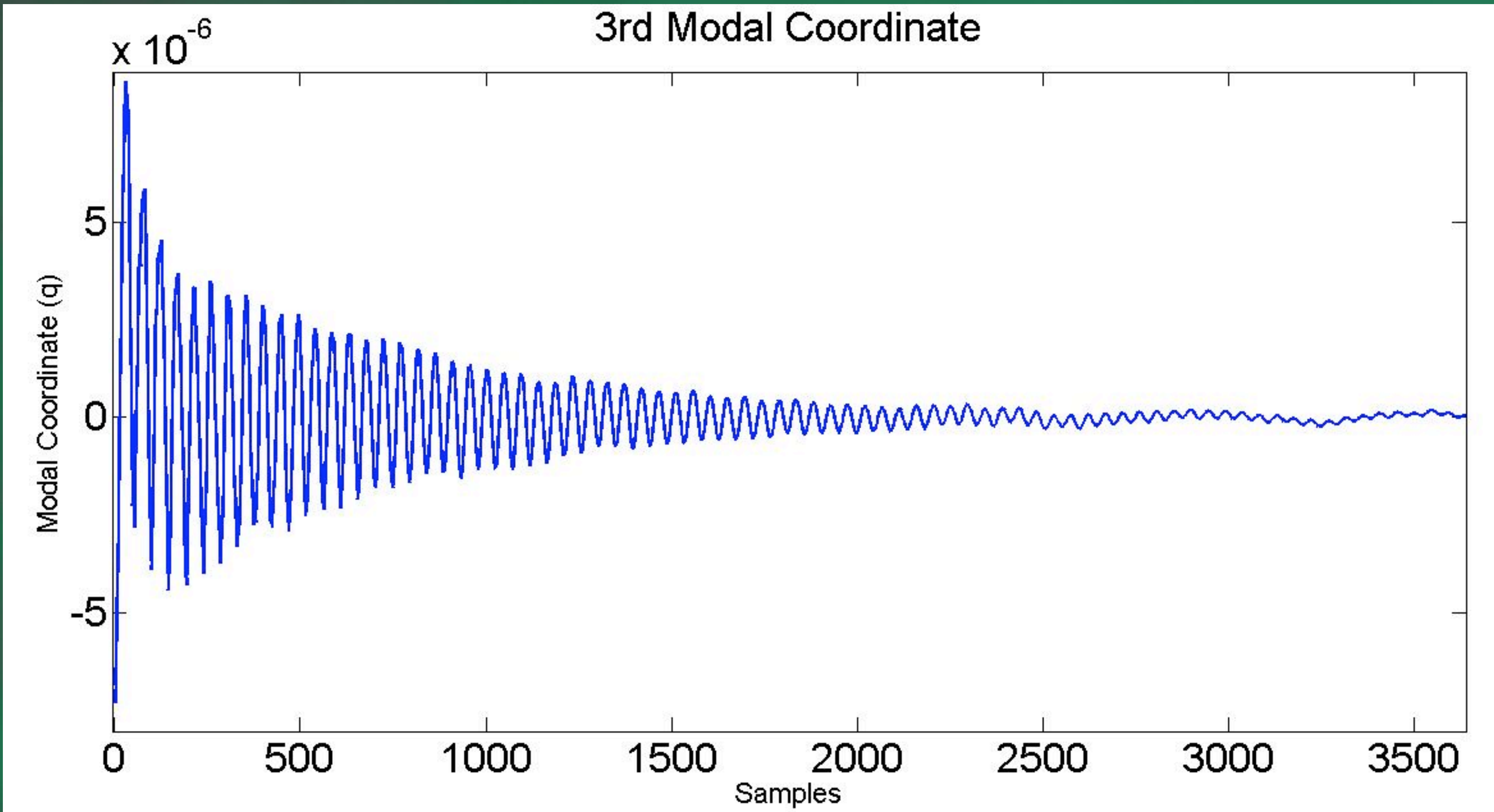

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Summary

- Reduced Mass Weighted POD eigenvectors provide approximations for linear normal modes.
- We applied this method to a beam experiment.

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THE END